

$$\text{Operating profit} = \text{Total revenues} - \text{Total costs}$$

$$\text{Profit} = \text{TR} - \text{TC}$$

$$\text{Total revenue} = \text{Price} \times \text{Units of output produced and sold}$$

$$\text{TR} = \text{PX}$$

$$\text{Total cost} = (\text{Variable costs per unit} \times \text{Units of output}) + \text{Fixed costs}$$

$$\text{TC} = \text{VX} + \text{F}$$

Therefore, Profit =  $\text{PX} - (\text{VX} + \text{F})$

Collecting terms gives:

$$\begin{aligned} \text{Profit} &= (\text{Price} - \text{Variable costs}) \times \text{Units of output} - \text{Fixed costs} \\ &= (\text{P} - \text{V})\text{X} - \text{F} \end{aligned}$$

Contribution margin per unit is defined as

$$\text{Price} - \text{Variable cost per unit}$$

$$\text{P} - \text{V}$$

Break-Even Volume in Units

We can use the profit equation to find the break-even point expressed in units:

$$\text{Profit} = 0 = (\text{P} - \text{V})\text{X} - \text{F}$$

$$\Rightarrow \text{X} = \frac{\text{F}}{\text{P} - \text{V}}$$

$$\text{Break-even volume (in units)} = \frac{\text{Fixed costs}}{\text{Unit Contribution margin}}$$

Contribution margin ratio

$$\text{Contribution margin ratio} = \frac{\text{Unit Contribution margin}}{\text{Sales price per unit}}$$

Break-even volume sales dollars

$$\text{Break-even volume sales dollars} = \frac{\text{Fixed costs}}{\text{Contribution margin ratio}}$$

Target Volume in Units

$$\text{Target volume (units)} = \frac{\text{Fixed costs} + \text{Target profit}}{\text{Contribution margin per unit}}$$

Target Volume in Sales Dollars

$$\text{Target volume (sales dollars)} = \frac{\text{Fixed costs} + \text{Target profit}}{\text{Contribution margin ratio}}$$

Margin of Safety

The margin of safety is the excess of projected (or actual) sales over the break-even sales level.

$$\text{Sales volume} - \text{Break-even sales volume} = \text{Margin of Safety}$$