Functional Dependencies

Theory

Example: $F = \{X \to Y, Y \to Z\}$ by definition of FDs, $X \to Z$ is "logically implied" by F.

Let F be a set of FDs. Let F^+ denote the closure of F, which is the set of all FDs logically implied by F.

Rules of Inference for FDs (FD rules)

1. Reflexivity if $Y \subseteq X$, then $X \to Y$
2. Augmentation if $X \to Y$, then $WX \to WY$
3. Transitivity if $X \to Y$ & $Y \to Z$, then $X \to Z$
4. Union if $X \to Y$ & $X \to Z$, then $X \to YZ$
5. Decomposition if $X \to YZ$, then $X \to Y$ & $X \to Z$
6. Pseudotransitivity if $X \to Y$ & $WY \to Z$, then $XW \to Z$

* Rules 1, 2 & 3 are Armstrong's axioms (complete)

* Rules 4, 5 & 6 follow from Armstrong's axioms

E.g. Consider a relation schema with attributes ABCGWXYZ and the set of functional dependencies $F = \{XZ \to ZYB, YA \to CG, C \to W, B \to G, XZ \to G\}$

(a) Is dependency $XZA \to YB$ implied by F.

This question is actually ask for does closure $(XZA)^+$ contains YB?

result:

X, Z, A	given
Y, B	$XZ \to ZYB$
⋮	

∴ Yes, $XZA \to YB$ is in F^+

(b) Is the decomposition into XZYAB and YABCGW lossless?

	A	B	C	G	W	X	Y	Z
$R_1 = (X,Z,Y,A,B)$	✓	✓	✓②	✓①	✓③	✓	✓	✓
$R_2 = (Y,A,B,C,G,W)$								

① $XZ \to G$

② $YA \to C, G$

③ $C \to W$

∵ there is a completed row

∴ This is lossless-join, that is R_1 and R_2 is lossless-join.

Method 2:

$R_1 \cap R_2 = \{Y, A, B\}$

if $(Y,A,B)^+ \Rightarrow R_1$ or R_2 ⇒ lossless.

$(Y,A,B)^+ = \{Y, A, B, C, G, W\}$ $YA \to CG$, $C \to W$

$= R_2$

∴ lossless-join decomposition