

The image shows the cover of a spiral-bound notebook. The cover is a light beige or tan color with a fine, woven texture. A silver metal spiral binding is visible along the left edge. The text is centered on the cover in a bold, black, serif font. The main title is "CSE 412/598 DATABASE MANAGEMENT COURSE NOTES" and the chapter title is "7. RELATIONAL DATABASE DESIGN". At the bottom, the department and university are listed.

**CSE 412/598**  
**DATABASE MANAGEMENT**  
**COURSE NOTES**

**7. RELATIONAL DATABASE DESIGN**

Department of Computer Science & Engineering  
Arizona State University

# RELATIONAL DATABASE DESIGN

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Goal: Generate a set of relation schemes that allow us to store information without unnecessary redundancy, yet allows us to retrieve information easily.

## Undesirable Properties

- Repetition of information
- Inability to represent certain information
- Loss of information

## Desirable Properties

- No Repetition of Information (Normal Forms)
- Dependency Preservation
- Lossless-join Decomposition

# UNDESIRABLE PROPERTIES

## Repetition of information

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suppliers(SNAME, SADDR, ITEM, PRICE)

The address of the supplier is repeated for each item supplied:

- ⇒ wasted space
- ⇒ every tuple for a supplier must be updated when there is an address change

# UNDESIRABLE PROPERTIES

## Inability to represent certain information

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suppliers(SNAME, SADDR, ITEM, PRICE)

⇒ Insertion:

we can't record an address for a supplier if that supplier does not currently supply at least one item.

⇒ Deletion:

if all the items supplied by one supplier are deleted, we lose the address.

# UNDESIRABLE PROPERTIES

## Loss of information

addresses(NAME, CITY, ZIP)  $\neq$

addr1(NAME, CITY)  $\bowtie$  addr2(CITY, ZIP)

The join of addr1 and addr2 is not equal to addresses

**addresses**  $\neq$  **addr1**  $\bowtie$  **addr2**

N C Z

-----  
bill phx 85050

jane phx 85051

N C

-----  
bill phx

jane phx

$\bowtie$

C Z

-----  
phx 85050

phx 85051

=

N C Z

-----  
bill phx 85050

bill phx 85051

jane phx 85050

jane phx 85051

# FUNCTIONAL DEPENDENCIES

## Definition

Recall superkey defn:

Let  $R$  be a relation scheme. A subset  $K$  of  $R$  is a superkey of  $R$  if, in any legal relation  $r(R)$ , for all pairs  $t_1$  and  $t_2$  tuples in  $r$  s.t.  $t_1 \neq t_2$

$$t_1[K] \neq t_2[K]$$

*"No two tuples in any legal relation  $r(R)$  may have the same value on attribute set  $K$ ".*

Let  $V \subseteq R$  and  $W \subseteq R$ . The **Functional Dependency** (FD)  $V \rightarrow W$  holds on  $R$  if in any legal relation  $r(R)$ , for all pairs of tuples  $t_1$  and  $t_2$  in  $r$  s.t.  $t_1[V] = t_2[V]$  it is also the case that  $t_1[W] = t_2[W]$ .

An FD  $X \rightarrow Y$  is **trivial** if  $Y \subseteq X$ .

Note: if  $K$  is a superkey of  $R$ , then  $K \rightarrow R$

# FUNCTIONAL DEPENDENCIES

## Examples

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1. suppliers(SNAME, SADDR, ITEM, PRICE)

$R = \{SNAME, SADDR, ITEM, PRICE\}$

$F = \{f_1: SNAME \rightarrow SADDR,$   
 $f_2: SNAME, ITEM \rightarrow PRICE\}$

2. addresses(NAME, CITY, ZIP)

$F = \{f_1: NAME \rightarrow ZIP,$   
 $f_2: ZIP \rightarrow CITY\}$

# MORE ON FUNCTIONAL DEPENDENCIES

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- A FD is a property of the meaning or semantics of the attributes in a relation schema.
- Legal relation instances  $r$  of  $R$  satisfy the functional dependency constraints.

FDs are used to

- provide a set of constraints on the attributes of a relation schema that must hold at all times
- provide additional information to be used in the design process



# FUNCTIONAL DEPENDENCIES

## Company Training Courses Enterprise

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Most of the FDs for an enterprise are already included in the conceptual model. Using the ER diagram and the semantics of the enterprise, give the FDs for the company training courses example.

# THEORY OF FUNCTIONAL DEPENDENCIES

Example:  $F = \{X \rightarrow Y, Y \rightarrow Z\}$

By definition of FDs,  $X \rightarrow Z$  is "logically implied" by  $F$ .

Let  $F$  be a set of FDs. Let  $F^+$  denote the closure of  $F$ , which is the set of all FDs logically implied by  $F$ .

## Rules of Inference for FDs (FD rules)

- |    |                    |   |
|----|--------------------|---|
| 1. | Reflexivity        | if $Y \subseteq X$ , then $X \rightarrow Y$                         |
| 2. | Augmentation       | if $X \rightarrow Y$ , then $WX \rightarrow WY$                     |
| 3. | Transitivity       | if $X \rightarrow Y$ & $Y \rightarrow Z$ , then $X \rightarrow Z$   |
| 4. | Union              | if $X \rightarrow Y$ & $X \rightarrow Z$ , then $X \rightarrow YZ$  |
| 5. | Decomposition      | if $X \rightarrow YZ$ , then $X \rightarrow Y$ & $X \rightarrow Z$  |
| 6. | Pseudotransitivity | if $X \rightarrow Y$ & $WY \rightarrow Z$ , then $XW \rightarrow Z$ |

- Rules 1, 2 & 3 are Armstrong's axioms (complete)
- Rules 4, 5 & 6 follow from Armstrong's axioms

# EXAMPLE PROOF OF INFERENCE

Let  $F = \{f_1: Z \rightarrow A, f_2: B \rightarrow X, f_3: AX \rightarrow Y\}$

Prove that  $ZB \rightarrow Y$  is in  $F^+$ ?

$ZB \rightarrow AB$

Augmentation( $f_1, B$ )

$AB \rightarrow AX$

Augmentation( $f_2, A$ )

$ZB \rightarrow AX$

Transitivity( $ZB \rightarrow AB, AB \rightarrow AX$ )

$ZB \rightarrow Y$

Transitivity( $ZB \rightarrow AX, f_3$ )

What if we just wanted to know whether a FD is in  $F^+$ ?

Computing  $F^+$  is costly. Rather than computing  $F^+$ , we can find the set of attributes functionally determined by the given set of attributes on the left-hand side of the FD to check.

# ATTRIBUTE CLOSURE

Let  $X$  be a set of attributes.

$X^+$  be the closure of  $X$  under  $F$ , i.e., the set of all attributes functionally determined by  $X$  under a set  $F$  of FDs.

Algorithm: compute  $X^+$

Input:  $F$  - set of FDs

$X$  - set of Attributes

Output:  $X^+$  - set of attributes

Result :=  $X$ ;

/\*reflexivity \*/

while (changes to Result) do

  for each FD  $Y \rightarrow Z$  in  $F$  do

    if  $Y \subseteq \text{Result}$  then Result := Result  $\cup$   $Z$ ;

/\* transitivity \*/

$X^+ := \text{Result}$ ;

## EXAMPLE: ATTRIBUTE CLOSURE

$F = \{$   
     $f_1: Z \rightarrow A,$   
     $f_2: B \rightarrow X,$   
     $f_3: AX \rightarrow Y\}$

Is  $ZB \rightarrow Y$  in  $F^+$ ?

Equivalently, is  $Y$  in  $(ZB)^+$ ?

Result:  $ZB,$   
         $A,$   
         $X,$   
         $Y$

Given  
     $f_1$   
     $f_2$   
     $f_3$

Yes!

# DECOMPOSITION

Let  $U$  be a relation scheme.

A set of relation schemes  $\{R_1, R_2, \dots, R_n\}$  is a **decomposition** of  $U$  if

$$\bigcup_{i=1}^n R_i = U$$

Let  $u$  be a relation on scheme  $U$ .

Let  $r_i = \pi_{R_i}(u)$ , for  $1 \leq i \leq n$

$$u \subseteq \bigotimes_{i=1}^n r_i$$

# DESIRABLE PROPERTIES

## Lossless-join Decomposition

Let  $C$  be a set of constraints on the DB. A decomposition  $\{R_1, R_2, \dots, R_n\}$  of a relation scheme  $U$  is a **lossless-join** decomposition if for all relations  $u$  on scheme  $U$  that are legal under  $C$

$$u = \bigjoin_{i=1}^n \Pi_{R_i}(u)$$

Let  $R$  be a relation scheme,  $F$  be a set of FDs on  $R$ ,  $R_1$  &  $R_2$  form a decomposition of  $R$ . This decomposition is a lossless-join decomposition of  $R$  if at least one of the following holds:

- $(R_1 \cap R_2) \rightarrow R_1$  in  $F^+$
- $(R_1 \cap R_2) \rightarrow R_2$  in  $F^+$

# LOSSLESS-JOIN DECOMPOSITION

## Suppliers Example

1. suppliers(SNAME, SADDR, ITEM, PRICE)  
 $R = \{SNAME, SADDR, ITEM, PRICE\}$   
 $F = \{f_1: SNAME \rightarrow SADDR, f_2: SNAME, ITEM \rightarrow PRICE\}$

Consider the decomposition  $\{R_1, R_2\}$  where

$$R_1 = \{SNAME, SADDR\}$$

$$R_2 = \{SNAME, ITEM, PRICE\}$$

$$R_1 \cap R_2 = \{SNAME\}$$

$$SNAME^+ = \{SNAME, SADDR\} = R_1$$

$\therefore$  lossless-join decomposition

$$r_1 = \text{suppaddr}(SNAME, SADDR)$$

$$r_2 = \text{suppitem}(SNAME, ITEM, PRICE)$$



# LOSSLESS-JOIN DECOMPOSITION

## Addresses Example

2. addresses(NAME, CITY, ZIP)  $F = \{f_1, f_2\}$

(a)  $f_1: \text{NAME} \rightarrow \text{CITY}$   
 $f_2: \text{ZIP} \rightarrow \text{CITY}$

Consider the decomposition

$$R_1 = \{\text{NAME}, \text{CITY}\}$$
$$R_2 = \{\text{CITY}, \text{ZIP}\}$$
$$R_1 \cap R_2 = \{\text{CITY}\}$$
$$(\text{CITY})^+ = \{\text{CITY}\}$$

$\therefore$  NOT a lossless-join decomposition

(b)  $f_1: \text{NAME} \rightarrow \text{ZIP}$   
 $f_2: \text{ZIP} \rightarrow \text{CITY}$

Consider the decomposition

$$R_1 = \{\text{NAME}, \text{ZIP}\}$$
$$R_2 = \{\text{CITY}, \text{ZIP}\}$$
$$R_1 \cap R_2 = \{\text{ZIP}\}$$
$$(\text{ZIP})^+ = \{\text{ZIP}, \text{CITY}\} = R_2$$

$\therefore$  lossless-join decomposition

# TESTING LOSSLESS-JOIN

INPUT:  $R$  = Relation Scheme, having  $j$  attributes  
 $D = \{R_1, \dots, R_i\}$  a decomposition of  $R$   
 $F$  = FDs on  $R$

1. create matrix  $S$  with  $i$  rows and  $j$  columns
2.  $S(i,j) := b_{ij}$ ; /\*  $b_{ij}$  a distinct symbol \*/
3. for each row  $i$  /\* representing a relation in  $D$  \*/  
for each column  $j$  /\* representing an attribute of  $R$  \*/  
if  $R_i$  includes attribute  $A_j$ , then  $S(i,j) := a_j$ ;
4. repeat until no changes  
for each FD  $X \rightarrow Y$  in  $F$   
for all rows in  $S$  which have the same symbols in the  
columns corresponding to attrs in  $X$   
make these rows have the same value in  $Y$   
(choose an “a” symbol over a “b” symbol)
5. lossless-join := exists a row having all “a” symbols;

# DESIRABLE PROPERTIES

## Dependency Preservation

"Update validation without computation of joins"

Let  $F$  be a set of FDs on a scheme  $R$ , and  $R_1, \dots, R_n$  be a decomposition of  $R$ . The restriction of  $F$  to  $R_i$  is the set  $F_i$  of all FDs in  $F^+$  that include only attrs of  $R_i$ . The set of restrictions  $F_1, F_2, \dots, F_n$  is the set of dependencies that can be checked efficiently. Let

$$F' = \bigcup_{i=1}^n F_i$$

In general,  $F \neq F'$ .

A dependency preserving decomposition has the property that  $(F')^+ = F^+$ , i.e., every dependency in  $F$  is logically implied by  $F'$ .

# TESTING DEPENDENCY PRESERVATION

compute  $F^+$

For each scheme  $R_i$  in  $D$  do

$F_i :=$  the restriction of  $F^+$  to  $R_i$ ;

$n$

$$F' := \bigcup_{j=1}^n F_j$$

compute  $(F')^+$

return  $((F')^+ = F^+)$

... Not always necessary to use this algorithm, we can check that each FD in  $F - F'$  is logically implied by  $F'$

# DEPENDENCY PRESERVATION

## Suppliers EXAMPLES

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1. suppliers(SNAME, SADDR, ITEM, PRICE)

$R = \{SNAME, SADDR, ITEM, PRICE\}$

$F = \{f_1: SNAME \rightarrow SADDR, f_2: SNAME, ITEM \rightarrow PRICE\}$

$R_1 = \{SNAME, SADDR\}$                        $F_1 = \{f_1\}$

$R_2 = \{SNAME, ITEM, PRICE\}$                $F_2 = \{f_2\}$

$F' = F,$

$\therefore$  dependency preserving

# DEPENDENCY PRESERVATION

## Addresses Example

2. addresses(NAME, CITY, ZIP)

(a)  $F = \{f_1: \text{NAME} \rightarrow \text{CITY}, f_2: \text{ZIP} \rightarrow \text{CITY}\}$

$R_1 = \{\text{NAME}, \text{ZIP}\} \quad F_1 = \emptyset$

$R_2 = \{\text{ZIP}, \text{CITY}\} \quad F_2 = \{f_2\}$

$F' = \{f_2\}$

$F - F' = \{f_1\}$

Is  $f_1$  logically implied by  $F'$  ?

No,  $\therefore$  NOT dependency preserving

(b)  $F = \{f_1: \text{NAME} \rightarrow \text{ZIP}, f_2: \text{ZIP} \rightarrow \text{CITY}\}$

$R_1 = \{\text{NAME}, \text{ZIP}\} \quad F_1 = \{f_1\}$

$R_2 = \{\text{CITY}, \text{ZIP}\} \quad F_2 = \{f_2\}$

$F' = F, \therefore$  dependency preserving

### Note:

1. A decomposition may have a lossless-join w.r.t.  $F$  yet not preserve  $F$ .
2. A decomposition could preserve  $F$  yet not have a lossless-join.

# NORMAL FORMS

## Boyce-Codd Normal Form (BCNF)

A relation scheme is in BCNF if for all FDs that hold on R of the form  $X \rightarrow Y$ , where  $X \subseteq R$  &  $Y \subseteq R$ , at least one of the following holds:

- $X \rightarrow Y$  is a trivial FD
- X is a superkey for scheme R

Example:  $R = \{EMP, PHONE, PROJECT, HRS\}$

$F = \{EMP \rightarrow PHONE, EMP \ PROJECT \rightarrow HRS, \\ PHONE \rightarrow EMP, PHONE \ PROJECT \rightarrow HRS\}$

candidate keys: EMP PROJECT and PHONE PROJECT

not BCNF: EMP/PHONE not superkey

BCNF:  $R_1 = \{\underline{EMP}, \underline{PHONE}\}$

$F_1 = \{EMP \rightarrow PHONE, \\ PHONE \rightarrow EMP\}$

$R_2 = \{\underline{EMP}, \underline{PROJECT}, HRS\}$

$F_2 = \{EMP, PROJECT \rightarrow HRS\}$

# BCNF DECOMPOSITION ALGORITHM

Let R be a given relation scheme, F be the given set of FDs on R.

Result := {R};

Done := FALSE;

compute  $F^+$ ;

while (not DONE) do

  if there is a scheme  $R_i$  in Result that is not in BCNF

    {Let  $X \rightarrow Y$  be a nontrivial FD on  $R_i$  s.t.

$X \rightarrow R_i$  is not in  $F^+$  /\* not already in BCNF \*/

      Result := (Result -  $R_i$ )  $\cup$  ( $R_i$  - Y)  $\cup$  (X  $\cup$  Y) }

  else

    DONE := TRUE;

**Note:** The order in which the Fds are chosen for decomposition affects the resulting schema.



# BCNF DECOMPOSITION ALGORITHM

## Guarantees Lossless-join property

$$\text{Result} := (\text{Result} - R_i) \cup (R_i - Y) \cup (X \cup Y)$$

The new relations added at each step are:

$$(R_i - Y) \ \& \ (X \cup Y)$$

Lossless-join check on new relations:

$$(R_i - Y) \cap (X \cup Y) = \{X\}$$

We know that  $X \rightarrow Y$ .

The attribute closure of X:  $(X)^+ \rightarrow XY$

functionally determines the scheme of  $(X \cup Y)$

$\therefore$  lossless-join decomposition

However, not every BCNF decomposition is dependency preserving.

# BCNF & DEPENDENCY PRESERVATION

## Counter-example

Let  $S = \{\underline{J}, \underline{K}, L\}$   
 $F = \{JK \rightarrow L, L \rightarrow K\}$   
S is not in BCNF since  $L \rightarrow K$  but L is not a superkey.  
 $(L)^+ = \{L, K\}$ , missing J.

BCNF decomposition of S using  $L \rightarrow K$ :

$S_1 = \{\underline{L}, K\} \quad L \rightarrow K$

$S_2 = \{\underline{J}, \underline{L}\}$

$S_1$  &  $S_2$  preserve only  $L \rightarrow K$  (& trivial dependencies)

but closure of  $\{L \rightarrow K\}$  does not include  $JK \rightarrow L$

$\therefore$  not dependency preserving, but BCNF and lossless-join