Functional Dependencies

Theory

Example: $F = \{X \to Y, Y \to Z\}$ by definition of FDs, $X \to Z$ is "logically implied" by F.

Let F be a set of FDs. Let F^+ denote the closure of F, which is the set of all FDs logically implied by F.

Rules of Inference for FDs (FD rules)

1. Reflexivity: if $Y \subseteq X$, then $X \to Y$
2. Augmentation: if $X \to Y$, then $WX \to WY$
3. Transitivity: if $X \to Y$ & $Y \to Z$, then $X \to Z$
4. Union: if $X \to Y$ & $X \to Z$, then $X \to YZ$
5. Decomposition: if $X \to YZ$, then $X \to Y$ & $X \to Z$
6. Pseudotransitivity: if $X \to Y$ & $WY \to Z$, then $XW \to Z$

* Rules 1, 2 & 3 are Armstrong's axioms (complete)

* Rules 4, 5 & 6 follow from Armstrong's axioms

E.g. Consider a relation schema with attributes ABCGWXYZ and the set of functional dependencies $F = \{XZ \to ZYB, YA \to CG, C \to W, B \to G, XZ \to G\}$

(a) Is dependency $XZA \to YB$ implied by F.

This question is actually ask for does closure $(XZA)^+$ contains YB?

result:

X, Z, A	given
Y, B	$XZ \to ZYB$
⋮	

∴ Yes, $XZA \to YB$ is in F^+

(b) Is the decomposition into XZYAB and YABCGW lossless?

	A	B	C	G	W	X	Y	Z
$R_1 = (X,Z,Y,A,B)$	✓	✓	✓ ②	✓ ①	✓ ③	✓	✓	✓
$R_2 = (Y,A,B,C,G,W)$								

① $XZ \to G$

② $YA \to C, G$

③ $C \to W$

∵ there is a completed row

∴ This is lossless-join, that is R_1 and R_2 is lossless-join.

Method 2:

$R_1 \cap R_2 = \{Y, A, B\}$

if $(Y,A,B)^+ \Rightarrow R_1$ or R_2 ⇒ lossless.

$(Y,A,B)^+ = \{Y, A, B, CG, W\}$

$YA \to CG$

$C \to W$

$= R_2$

∴ lossless-join decomposition

Functional Dependencies

Suppose that we have the following three tuples in $S(A,B,C) = (1,2,3), (4,2,3), (5,3,3)$

Which of the following dependencies can you infer does not hold over schema S?

(i) $A \to B$ (ii) $BC \to A$ (iii) $B \to C$

S

A	B	C
1	2	3
4	2	3
5	3	3

(i) $A \to B$ holds over S ☆(Which means we can define $S(A)=B$ which is a function)

(ii) does not hold over S, because $(1,2,3)$ and $(4,2,3)$ have same value for BC but different value for A. As a Function same left hand side could not have different right hand side value. $y=f(x)$ a x value could not be mapped to more than 1 x value.

(iii) $B \to C$ holds over S.

Find keys KEY → Everything

Consider a relation $R(A,B,C,D,E,F)$ with the following set of functional dependencies:

$A \to C$, $DE \to F$, $B \to D$

What is the key for R?

$\{A,B,E\}^+ = \{A,B,E,C,D,F\} = R \Rightarrow \{A,B,E\}$ is the key for R.

Example of lossless

Consider the schema $R = ABCDE$ with the set of FDs $F = \{AB \to CD, C \to B\}$. Is the decomposition $\{AE, BC, ACD\}$ of R lossless with respect to F?

init:

	A	B	C	D	E
AE	a				e
BC		b	c		
ACD	a		c	d	

applying $C \to B$:

	A	B	C	D	E
AE	a				e
BC		b	c		
ACD	a	b	c	d	

applying $AB \to CD$

nothing change.

$\because$ we have no rows complete with a, b, c, d, e the decomposition is lossy.